CSC 363, Winter 2010 — Short Assignment #1

*Due at **start** of tutorial time on January 20. Late assignments will **not** be accepted, as the tutorial instructors will immediately go over the solution.*

This assignment is to be done by each student individually. You are encouraged to discuss the course material in general with other students, but you should not discuss this assignment (verbally, in writing, by email, or in any other way) with people other than the course instructor and tutors, except to clarify the meaning of the question. Handing in work that is not your own is a serious academic offense.

We'd like to use a deterministic Turing Machine (TM) to decide the language, L, on the alphabet $\Sigma = \{a, b, c\}$, that consists of all strings in which the number of occurrences of a is at least as large as the number of occurrences of b. For example, *cabbbacaa* and *aabbcbaba* are in L, but *ababcb* and *cccb* are not. The empty string is in L.

We could create a TM that actually counts the a and b symbols, recording the count as a binary number, and then compares the two counts, but that would require programing the TM to do arithmetic. It's easier to just wipe out pairs of a and b symbols until either there are no b symbols left, in which case we accept, or there are no a symbols (but still at least one b symbol), in which case we reject.

Here is an informal description of a TM that uses this method to decide the language L:

M = "On input w,

1. Scan the tape moving toward the right until one of the following:
   - A blank is reached, in which case *accept*.
   - An a is reached, in which case replace it with a c and go to stage 2.
   - A b is reached, in which case replace it with a c and go to stage 3.
2. Continue scanning the tape moving right until one of the following:
   - A blank is reached, in which case *accept*.
   - A b is reached, in which case replace it with a c and go to stage 4.
3. Continue scanning the tape moving right until one of the following:
   - A blank is reached, in which case *reject*.
   - An a is reached, in which case replace it with a c and go to stage 4.
4. Move the head back to the start of the tape, then go to stage 1.

Write a complete formal description of a Turning Machine M that follows this informal description (a diagram such as in Figures 3.8 or 3.10 would be fine). The tape alphabet, Γ, the finite set of states, Q, and the transition function, δ, are up to you to choose, subject to the constraints in the definition of a TM on page 140 of Sipser's text. Note that stage 4 is a bit tricky. See the middle of page 144 for an idea regarding that. You might want to introduce "marked" versions of a, b, and c into the tape alphabet to help here.