STA 247 — Assignment #1, Due 4pm October 20

(Hand in your assignment in lecture, or in SS 6016A)

Please submit your assignment on 8 1/2 *by* 11 *inch paper, stapled in the upper-left corner. Do* ***not*** *put it in an envelope or folder. Put your name, student number, and lecture section (Day or Evening) on the first page.*

Late assignments will be accepted only with a valid medical or other excuse.

This assignment is to be done by each student individually.

For all questions, show how you obtained your answer. Numerical answers must be given explicitly, as a simple fraction or decimal number.

Question 1: Suppose you throw three six-sided dice (coloured red, green, and blue) repeatedly, until the three dice all show different numbers. Assuming that these dice are equally likely to show any combination of three different numbers, calculate the probabilities of the following events:

a) The green die shows the number 6.

b) The sum of the numbers shown by the three dice is 6.

c) The sum of the numbers shown by the three dice is at least 14.

d) All three dice show odd numbers.

Question 2: The conditional probability $P(C|A)$ represents how likely the event C is if we already know that the event A has occurred (and we know nothing else). If $P(C|A) > P(C)$, the occurrence of A increases our belief that C has occurred. If $P(C|B) > P(C)$ as well, we might expect that if we know that **both** A and B have occurred, our belief that C has occurred would be at least as great as when we know only that A occurred or only that only B has occurred. This intuition leads to the following conjecture:

> **Conjecture:** Let A, B, and C be any three events, with $P(A) > 0$, $P(B) > 0$, and $P(A \cap B) > 0$. If $P(C|A) > P(C)$ and $P(C|B) > P(C)$, then $P(C|A \cap B) > P(C)$.

EITHER prove that this conjecture is true, using the three basic axiom of probability (p. 8 in the text) and the definition of conditional probability (p. 27), **OR** show that the conjecture is not true, by finding a specific example of a sample space, S, probabilities for outcomes in S, and events A, B, and C for which the conjecture is false.

Question 3: You flip a coin. If the coin lands heads, you roll two six-sided dice. If the coin lands tails, you roll one six-side die. Define the following random variables:

X = 1 if the coin lands tails, 2 if the coin lands heads

Y = The sum of the numbers on all the dice rolled (either one or two dice)

Assuming that the coin and dice are fair,

a) Write down a table showing the joint probability mass function for X and Y.

b) Write down a table showing the marginal probability mass function for Y.

c) Write down a table showing conditional probability mass function for X given $Y = 5$.

Question 4: A software development firm has two programmers, Alice and Jill, to whom they assign software programming projects at random, with equal probabilities. Alice is a rather good programmer. Half the time, her programs have no bugs, and otherwise they have just a single bug. Jill isn't as good. She has an equal chance of producing a program with no bugs, one bug, two bugs, three bugs, or four bugs (ie, each of these possibilities has probability 1/5). After Alice or Jill finish a program, it is sent to the testing department, where they try to find the bugs. The testers have a 1/3 chance of finding each bug, and whether they find one bug is independent of whether they find any other bug (ie, the events of them finding each bug are mutually independent).

a) What is the expected number of bugs in a program (which might be written by either Alice or Jill) that is sent to the testers?

b) What is the expected number of bugs in a program that are not found by the testers?

c) Suppose the testers find exactly one bug in a program. What is the probability that there are no more bugs in this program? What is the probability that this program was written by Alice?