Question 1: The random variable $X$ has a range of $\{0, 1, 2\}$ and the random variable $Y$ has a range of $\{1, 2\}$. The joint distribution of $X$ and $Y$ is given by the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$P(X = x, Y = y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

1. Write down tables for the marginal distributions of $X$ and of $Y$, i.e. give the values of $P(X = x)$ for all $x$, and of $P(Y = y)$ for all $y$.

2. Write down a table for the conditional distribution of $X$ given that $Y = 2$, i.e. give the values of $P(X = x | Y = 2)$ for all $x$.

3. Compute $E(X)$ and $E(Y)$.

4. Compute $E(XY)$.

5. Are $X$ and $Y$ independent? Explain why or why not.

Question 2: You roll one red die and one green die. Define the random variables $X$ and $Y$ as follows:

$X =$ The number showing on the red die

$Y =$ The number of dice that show the number six

For example, if the red and green dice show the numbers 6 and 4, then $X = 6$ and $Y = 1$.

Write down a table showing the joint probability mass function for $X$ and $Y$, find the marginal distribution for $Y$, and compute $E(Y)$.

Question 3. Suppose you roll two fair, six-sided dice, one of which is red and the other of which is green. Define the following random variables:

$X =$ The number shown on the red die

$Y =$ \begin{cases} 
0 & \text{if the two dice show the same number} \\
1 & \text{if the number on the green die is bigger than the number on the red die} \\
2 & \text{if the number on the red die is bigger than the number on the green die} 
\end{cases}

a) Write down a table showing the joint probability mass function for $X$ and $Y$.

b) Find the marginal probability mass function for $Y$, and compute its expected value.

c) Find the conditional probability mass function for $X$ given $Y = 1$. 

Question 4. You flip a fair coin. If the coin lands heads, you roll a fair six-sided die 100 times. If the coin lands tails, you roll the die 101 times. Let $X$ be 1 if the coin lands heads and 0 if the coin lands tails. Let $Y$ be the total number of times that you roll a 6. Find $P(X = 1 | Y = 15)$.

Question 5. Suppose you randomly pick an integer from 1 to 3, with the three possibilities being equally likely. Call this integer $N$. You then randomly pick an integer from $N$ to 3, with the $4 - N$ possibilities being equally likely. Call this second integer $M$. What is the probability that $M$ will be 3?

Question 6. Three computers, A, B, and C, are linked by network connections as shown below:

![Diagram of network connections between A, B, and C]

Two network connections, a1 and a2, link computer A and computer B. Three network connections, b1, b2, and b3, link computer B and computer C. Since there is no direct connection from computer A to computer C, messages sent from computer A to computer C must pass through computer B.

When computer A sends a message to computer B, it randomly chooses whether to use connection a1 or connection a2, with probabilities of $2/3$ for a1 and $1/3$ for a2. When computer B sends a message to computer C, it randomly chooses whether to use connection b1, connection b2, or connection b3, with probabilities of $1/2$ for b1, $1/4$ for b2, and $1/4$ for b3.

The time to send a message through each connection is 1ms for a1, 2ms for a2, 1ms for b1, 2ms for b2, and 3ms for b3. When a message is sent from computer A to computer C through computer B, it takes no time for computer B to take the message received from connection a1 or a2 and send it on connection b1, b2, or b3.

Let $X$ be the random variable whose value is the total time (in milliseconds) taken for a message sent from computer A to computer C to arrive.

a) Find $P(X = 4)$.

b) Find $E(X)$.

c) Suppose that a message sent from computer A to computer C takes 4ms or more to arrive (ie, $X \geq 4$). How likely is it that this message was sent from computer B to computer C using connection b3?