STA 247 — Quiz #4, 2001-11-11, 3:10pm – 35 minutes long

No books, no notes, and no calculators may be used.

All numerical answers must be actual numbers (decimals such as 0.15 or simple fractions such as 3/13 or 4/3), not just a formula. If this requires arithmetic on numbers bigger than 1000, you’ve either made a mistake, or you should think of an easier way to solve the problem.

**Q1:** [45 marks] You are a participant in a game show, where you are shown an urn containing 100 black balls and 200 white balls. The game show host will randomly draw 72 balls from this urn, *replacing each ball before drawing the next*, and will pay you $1000 if the number of times a black ball is drawn is 30 or less (if not, you get nothing). Find the probability that you will win the $1000. An good approximate answer is sufficient. Show your work.

You may find the following table of the cumulative distribution function of the standard normal distribution to be helpful:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3.0$</th>
<th>$-2.5$</th>
<th>$-2.0$</th>
<th>$-1.5$</th>
<th>$-1.0$</th>
<th>$-0.5$</th>
<th>$0.0$</th>
<th>$+0.5$</th>
<th>$+1.0$</th>
<th>$+1.5$</th>
<th>$+2.0$</th>
<th>$+2.5$</th>
<th>$+3.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X \leq x)$</td>
<td>0.001</td>
<td>0.006</td>
<td>0.023</td>
<td>0.067</td>
<td>0.159</td>
<td>0.309</td>
<td>0.500</td>
<td>0.691</td>
<td>0.841</td>
<td>0.933</td>
<td>0.977</td>
<td>0.994</td>
<td>0.999</td>
</tr>
</tbody>
</table>
Q2: [55 total marks] Here is a plot of the cumulative distribution function for a random variable $X$:

![Cumulative Distribution Function Plot](image)

a) [25 marks] Draw a plot of the probability density function for $X$.

b) [10 marks] Find the numerical value of $P(-3 < X < 0)$. Explain briefly how you got your answer.

c) [10 marks] Find the numerical value of $E(X)$. Show your work.

d) [10 marks] Define $Y = X/3$. Find the numerical value of the probability density of $Y$ at the point $-1$. In other words, find $f_Y(-1)$. Show your work.