STA 3000, Problem Set 4. Due in class December 5.

1. Chapter 3, exercise 12.
2. Chapter 3, exercise 14. Hint for (a): Find a parameterized family of decision rules that includes $\delta(x) = x$, and also includes a rule that dominates it.
3. Chapter 3, exercise 17.
4. Do the following:

   (a) Show that if $\lambda_1 \ll \lambda_2$ and $\lambda_2 \ll \lambda_1$, then any decision rule that is $\lambda_1$-admissible is also $\lambda_2$-admissible.

   (b) Give an example of a decision rule $\delta$ and measures $\lambda_1$ and $\lambda_2$ for which $\lambda_1 \ll \lambda_2$ and $\delta$ is $\lambda_1$-admissible, but $\delta$ is not $\lambda_2$-admissible.

   (c) Give an example of a decision rule $\delta$ and measures $\lambda_1$ and $\lambda_2$ for which $\lambda_2 \ll \lambda_1$ and $\delta$ is $\lambda_1$-admissible, but $\delta$ is not $\lambda_2$-admissible.

5. Suppose a positive integer $X$ is observed from a distribution of the following form:

$$P_\theta(X = x) = \begin{cases} 3/4 & \text{if } x = \theta \\ 1/4 & \text{if } x = \theta + 1 \\ 0 & \text{otherwise} \end{cases}$$

   where the parameter $\theta$ is a positive integer (ie, $\Omega = \{1, 2, 3, \ldots\}$). We wish to find a decision rule that estimates $\theta$ with $0-1$ loss — ie,

$$L(\theta, \delta(x)) = \begin{cases} 0 & \text{if } \delta(x) = \theta \\ 1 & \text{if } \delta(x) \neq \theta \end{cases}$$

   (1) Find the risk functions for each of the estimators $\delta_1(x) = x$, $\delta_2(x) = x + 1$, and $\delta_3(x) = x - 1$.

   (2) Find a Bayes rule for each of the following prior distributions:

   (a) $P(\Theta = \theta) = 2^{-\theta}$

   (b) $P(\Theta = \theta) = 3 \cdot 4^{-\theta}$

   (c) $P(\Theta = \theta) = 2 \cdot 3^{-\theta}$

   Determine whether or not each of these rules is admissible.

   (3) Which of the rules $\delta_1(x) = x$, $\delta_2(x) = x + 1$, and $\delta_3(x) = x - 1$ are admissible?

   (4) Find a minimax rule for this decision problem.