Question 1: Consider a decision problem in which the data space, parameter space, and action space are all the set of real numbers. The distribution of the data, $X$, given the parameter, $\theta$, is $N(\theta, 1)$. The loss function is $L(\theta, a) = (a - \theta)^2$.

Consider decision rules of the form $\delta_{a,b}(x) = ax + b$, where $a$ and $b$ can be any real numbers.

a) Find a simple formula for the risk function, $R(\theta, \delta_{a,b})$ for all $a$ and $b$.

b) Which rules $\delta_{a,b}$ are dominated by some other rule of this form (i.e., by $\delta_{a',b'}$ for some $a'$ and $b'$)?

c) For each of the rules of this form that are not dominated by another of this form, can you find a prior distribution for $\theta$ for which this rule is a formal Bayes rule?

d) Say what you can about the admissibility of each of the rules $\delta_{a,b}$.

Question 2: Consider a decision problem in which the data space, parameter space, and action space are all the set of positive real numbers. The distribution of the data, $X$, given the parameter, $\theta$, is $U(0, 1 + \theta)$. We use a prior for which the probability density is proportional to $(1 + \theta)^{-r}$, where $r$ is a fixed constant greater than one. We use the loss function $L(\theta, a) = (\theta - a)^2$.

a) For what values of $r$ does a formal Bayes rule, $\delta_r$, exist for all $x$? For those values of $r$, find a formal Bayes rule, and find a formula for the posterior risk, $r(\delta|x)$, of this rule.

b) For each formal Bayes rule, $\delta_r$, that you found in (a), find the risk function, $R(\theta, \delta_r)$.

c) For what values of $r$ is the formal Bayes rule, $\delta_r$, that you found in (a) also a Bayes rule?

d) Say what you can about the admissibility of the formal Bayes rules you found in (a).