Question 1: Consider a model in which both the data space, $\mathcal{X}$, and the parameter space, $\Omega$, are finite sets, and in which $P_\theta(\{x\}) > 0$ for all $\theta \in \Theta$ and $x \in \mathcal{X}$. Let $T(X)$ be some statistic. Prove that if the cardinality of the range of $T(X)$ is greater than the cardinality of $\Omega$, then $T(X)$ is not a complete statistic.

Question 2: Given $\theta \in (0, \infty)$, suppose that for a fixed $n$ that is known, $Y_1, \ldots, Y_n$ are IID observations with an exponential distribution with mean $\theta$ shifted to the right by $\theta$. In other words, the density for each $Y_i$ is $I(y > \theta)(1/\theta)\exp(-(y - \theta)/\theta)$. Find the minimal sufficient statistic, and show that it is not complete by finding two different unbiased estimators of $\theta$ that are both functions of the minimal sufficient statistic.